Compito A

4.

a)

$$\frac{1}{2}\cancel{M}V^2 = \frac{1}{2}\cancel{M}V_0^2 - \cancel{M}gh$$

$$V^2 = V_0^2 - 2gh = 75^2 - 2\cdot 9.8\cdot 285.15 = 5625 - 5588{,}94 = 36{,}06$$

$$V = 6.0ms^{-1}$$

$$\vec{v}_2 = \vec{v}_1 \wedge \hat{k} = \det\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 0\\ 0 & 0 & 1\end{vmatrix} = 2\cdot\hat{i} - 3\cdot\hat{j} + 0\cdot\hat{k} = 2\hat{i} - 3\hat{j}$$

b)

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\begin{cases} 4\cdot 3 + 2\cdot 2 + 4\cdot v_{3x} = MV_x = 10\cdot 0 = 0 \\ 4\cdot 2 + 2\cdot(-3) + 4\cdot v_{3y} = MV_y = 10\cdot 0 = 0 \\ 4\cdot 0 + 2\cdot 0 + 4\cdot v_{3z} = MV_z = 10\cdot 6 = 60 \end{cases}$$

$$v_{3x} = -\frac{16}{4} = -4$$

$$v_{3y} = -\frac{1}{2}$$

$$v_{3z} = \frac{60}{4} = 15$$

Compito A

5.

$$\vec{F}(x,y,z) = -\alpha\left[3x^2 z\,\hat{i} + 4yz^2\,\hat{j} + (x^3 + 4y^2 z)\hat{k}\right]$$

$$\frac{\partial F_x}{\partial y} = 0 \qquad \frac{\partial F_y}{\partial x} = 0$$

$$\frac{\partial F_x}{\partial z} = 3x^2 \qquad \frac{\partial F_z}{\partial x} = 3x^2$$

$$\frac{\partial F_y}{\partial z} = 8yz \qquad \frac{\partial F_z}{\partial y} = 8yz$$

$$U(x,y,z) = -\alpha\left(x^3 z + 2y^2 z^2\right)$$

$$U(0,0,0) = -\alpha \cdot 0 = 0J$$

$$U(A) = U(5,0,5) = -2 \cdot 625 = -1250J$$

$$L_{OA} = U(A) - U(O) = -1250J$$

Compito A

6.

a)

$$\vec{K}_{palla} = \vec{d} \wedge \vec{p} = \vec{d} \wedge m\vec{v} = 0.60 \cdot 2 \cdot 7 = 8.4\,kgm^2 s^{-1}$$

b)

$$\vec{K}_{palla} = \vec{K}_{piattaforma+persona}$$

$$\vec{K}_{piattaforma+persona} = 8.4\,kgm^2 s^{-1} = I\omega$$

$$I = \frac{1}{2}MR^2 + m_{persona}d^2 = \frac{1}{2}200 + 80 \cdot 0.36 = 100 + 28.8 = 128.8\,kgm^2$$

$$\omega = \frac{8.4}{128.8} = 0.0652\,rad/s$$

Compito A

7.

$$0 = m_t v_t - m_r v_{0x}$$

$$v_t = \frac{m_r}{m_t} v_r$$

$$-\frac{1}{2} g t^2 + v_{0y} t = 0$$

$$t_1 = 0$$

$$-\frac{1}{2} g t + v_{0y} = 0$$

$$t_2 = \frac{2v_{0y}}{g}$$

$$x_r = v_{0x} t_2 = \frac{2v_{0x} v_{0y}}{g}$$

$$x_t = v_t t_2 = \frac{m_r}{m_t} \frac{2v_{0x} v_{0y}}{g}$$

$$x_r + x_t = l$$

$$l = \frac{2v_{0x} v_{0y}}{g} + \frac{m_r}{m_t} \frac{2v_{0x} v_{0y}}{g} = \frac{2}{g} v_0^2 \sin 15° \cos 15° \left(1 + \frac{m_r}{m_t}\right)$$

$$v_0^2 = \frac{gl}{2 \sin 15° \cos 15°} \frac{m_t}{m_t + m_r} = \frac{9.8 \cdot 2}{2 \cdot 0.25} \frac{4}{4.1} = 38.24$$

$$v_0 = 6.18 ms^{-1}$$

Compito B

4.
a)

$$\vec{v}_3 = \vec{v}_1 \wedge \hat{i} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0 \cdot \hat{i} - 0 \cdot \hat{j} + (-3)\hat{k} = -3\hat{k}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\begin{cases} 3 \cdot 1 + 5 \cdot v_{2x} + 4 \cdot 0 = MV_x = 12 \cdot 3 = 36 \\ 3 \cdot 3 + 5 \cdot v_{2y} + 4 \cdot 0 = MV_y = 12 \cdot (-2) = -24 \\ 3 \cdot 0 + 5 \cdot v_{2z} - 4 \cdot 3 = MV_z = 12 \cdot 4 = 48 \end{cases}$$

$$v_{2x} = \frac{33}{5}$$

$$v_{2y} = -\frac{33}{5}$$

$$v_{2z} = 12$$

b)

$$\frac{1}{2}\cancel{M}V^2 = \cancel{M}gh$$

$$h = \frac{V^2}{2g} = \frac{16}{2 \cdot 9.8} = 0.82m$$

Compito B

5.

$$\vec{F}(x,y,z) = -\alpha\left[3x^2 y\hat{i} + \left(x^3 + 4yz^2\right)\hat{j} + 4y^2 z\hat{k}\right]N$$

$$\frac{\partial F_x}{\partial y} = 3x^2 \qquad \frac{\partial F_y}{\partial x} = 3x^2$$

$$\frac{\partial F_x}{\partial z} = 0 \qquad \frac{\partial F_z}{\partial x} = 0$$

$$\frac{\partial F_y}{\partial z} = 8yz \qquad \frac{\partial F_z}{\partial y} = 8yz$$

$$U(x,y,z) = -\alpha\left(x^3 y + 2y^2 z^2\right)$$

$$U(0,0,0) = -\alpha \cdot 0 = 0J$$

$$U(5,0,5) = -\alpha \cdot 0 = 0J$$

$$L_{OA} = U(A) - U(O) = 0J$$

Compito B

6.
a)

$$\vec{K}_{palla} = \vec{d} \wedge \vec{p} = \vec{d} \wedge m\vec{v} = 0.80 \cdot 1 \cdot 9 \cdot \frac{\sqrt{2}}{2} = 3.6\sqrt{2}kgm^2s^{-1} = 5{,}09\,kgm^2s^{-1}$$

b)

$$\vec{K}_{palla} = \vec{K}_{piattaforma+palla+persona}$$
$$\vec{K}_{piattaforma+palla+persona} = 5{,}09\,kgm^2s^{-1} = I\omega$$
$$I = \frac{1}{2}MR^2 + m_{persona+palla}d^2 = \frac{1}{2}250 \cdot 4 + 76 \cdot 0.64 = 500 + 48.64 = 548.64\,kgm^2$$
$$\omega = \frac{5{,}09}{548.64} = 0.00928\,rad/s$$

Compito B

7.

$$0 = m_t v_t - m_r v_{0x}$$

$$v_t = \frac{m_r}{m_t} v_r$$

$$-\frac{1}{2} g t^2 + v_{0y} t = 0$$

$$t_1 = 0$$

$$-\frac{1}{2} g t + v_{0y} = 0$$

$$t_2 = \frac{2 v_{0y}}{g}$$

$$x_r = v_{0x} t_2 = \frac{2 v_{0x} v_{0y}}{g}$$

$$x_t = v_t t_2 = \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g}$$

$$x_r + x_t = l$$

$$l = \frac{2 v_{0x} v_{0y}}{g} + \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g} = \frac{2}{g} v_0^2 \sin 15° \cos 15° \left(1 + \frac{m_r}{m_t}\right)$$

$$l = \frac{2}{9.8} (2.5)^2 \sin 15° \cos 15° \left(1 + \frac{0.2}{3}\right) = \frac{2}{9.8} 6.25 \cdot 0.25 \frac{3.2}{3} = 0.34 m$$

Compito C

4.
a)

$$\vec{v}_1 = \vec{v}_2 \wedge \hat{i} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 6 & 12 \\ 1 & 0 & 0 \end{vmatrix} = 0 \cdot \hat{i} - (-12)\hat{j} + (-6)\hat{k} = 12\hat{j} - 6\hat{k}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\begin{cases} 4 \cdot 0 + 2 \cdot (-4) + 5 \cdot v_{3x} = MV_x = 11 \cdot 2 = 22 \\ 4 \cdot 12 + 2 \cdot 6 + 5 \cdot v_{3y} = MV_y = 11 \cdot 3 = 33 \\ 4 \cdot (-6) + 2 \cdot 12 + 5 \cdot v_{3z} = MV_z = 11 \cdot 6 = 66 \end{cases}$$

$$v_{3x} = 6$$

$$v_{3y} = \frac{33 - 60}{5} = -\frac{27}{5}$$

$$v_{3z} = \frac{66 + 24 - 24}{5} = \frac{66}{5}$$

b)

$$\frac{1}{2}\cancel{M}V^2 = \cancel{M}gh$$

$$h = \frac{V^2}{2g} = \frac{36}{2 \cdot 9.8} = 1.84m$$

Compito C

5.

$$\vec{F}(x,y,z) = -\alpha\left[\left(y^3 + 4xz^2\right)\hat{i} + 3xy^2\hat{j} + 4x^2z\hat{k}\right]N$$

$$\frac{\partial F_x}{\partial y} = 3y^2 \qquad \frac{\partial F_y}{\partial x} = 3y^2$$

$$\frac{\partial F_x}{\partial z} = 8xz \qquad \frac{\partial F_z}{\partial x} = 8xz$$

$$\frac{\partial F_y}{\partial z} = 0 \qquad \frac{\partial F_z}{\partial y} = 0$$

$$U(x,y,z) = -\alpha\left(y^3x + 2x^2z^2\right)$$

$$U(0,0,0) = -\alpha \cdot 0 = 0J$$

$$U(A) = U(4,0,4) = -3(2 \cdot 16 \cdot 16)J = -3 \cdot 512J = -1536J$$

$$L_{OA} = U(A) - U(O) = -1536J$$

Compito C

6.

a)

$$\vec{K}_{palla} = \vec{d} \wedge \vec{p} = \vec{d} \wedge m\vec{v} = 0.60 \cdot \cancel{2} \cdot 7 \cdot \frac{\sqrt{3}}{\cancel{2}} = 4.2\sqrt{3} kgm^2 s^{-1} = 7,27 kgm^2 s^{-1}$$

b)

b)

$$\vec{K}_{palla} = \vec{K}_{piattaforma+palla+persona}$$

$$\vec{K}_{piattaforma+palla+persona} = 7,27 kgm^2 s^{-1} = I\omega$$

$$I = \frac{1}{2}MR^2 + m_{persona+palla}d^2 = \frac{1}{2}200 + 82 \cdot 0.36 = 100 + 29.52 = 129.52 kgm^2$$

$$\omega = \frac{7,27}{129.52} = 0.0562 rad/s$$

Compito C

7.

$$0 = m_t v_t - m_r v_{0x}$$

$$v_t = \frac{m_r}{m_t} v_r$$

$$-\frac{1}{2} g t^2 + v_{0y} t = 0$$

$$t_1 = 0$$

$$-\frac{1}{2} g t + v_{0y} = 0$$

$$t_2 = \frac{2 v_{0y}}{g}$$

$$x_r = v_{0x} t_2 = \frac{2 v_{0x} v_{0y}}{g}$$

$$x_t = v_t t_2 = \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g}$$

$$x_r + x_t = l$$

$$l = \frac{2 v_{0x} v_{0y}}{g} + \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g} = \frac{2}{g} v_0^2 \sin 30° \cos 30° \left(1 + \frac{m_r}{m_t}\right)$$

$$l = \frac{2}{9.8} 2^2 \sin 30° \cos 30° \left(1 + \frac{0.15}{4}\right) = \frac{2}{9.8} 4 \frac{\sqrt{3}}{4} \frac{4.15}{4} = 0.367 m$$

Compito D

4.
a)

$$\frac{1}{2}\cancel{M}V_0^2 = \frac{1}{2}\cancel{M}V^2 + \cancel{M}gh$$

$$V^2 = V_0^2 - 2gh = 83^2 - 2\cdot 9.8\cdot 292.75 = 6889 - 5737.9 = 1151.1$$

$$V = 33.93 ms^{-1}$$

b)

$$\vec{v}_3 = \vec{v}_2 \wedge \hat{j} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & 4 \\ 0 & 1 & 0 \end{vmatrix} = -4\hat{i} - 0\cdot\hat{j} + 0\cdot\hat{k} = -4\hat{i}$$

$$M\vec{V} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\begin{cases} 5\cdot v_{1x} + 4\cdot 0 + 3\cdot(-4) = MV_x = 12\cdot 0 = 0 \\ 5\cdot v_{1y} + 4\cdot(-3) + 3\cdot 0 = MV_y = 12\cdot 0 = 0 \\ 5\cdot v_{1z} + 4\cdot 4 - 3\cdot 0 = MV_z = 12\cdot 33.93 = 407.16 \end{cases}$$

$$v_{1x} = \frac{12}{5}$$

$$v_{1y} = \frac{12}{5}$$

$$v_{1z} = \frac{391.16}{5}$$

Compito D

5.

$$\vec{F}(x,y,z) = -\alpha\left[4xz^2\hat{i} + 3y^2z\hat{j} + \left(y^3 + 4x^2z\right)\hat{k}\right]N$$

$$\frac{\partial F_x}{\partial y} = 0 \qquad \frac{\partial F_y}{\partial x} = 0$$

$$\frac{\partial F_x}{\partial z} = 8xz \qquad \frac{\partial F_z}{\partial x} = 8xz$$

$$\frac{\partial F_y}{\partial z} = 3y^2 \qquad \frac{\partial F_z}{\partial y} = 3y^2$$

$$U(x,y,z) = -\alpha\left(2x^2z^2 + y^3z\right)$$

$$U(0,0,0) = -\alpha \cdot 0 = 0J$$

$$U(A) = U(6,0,6) = -(-1)(2\cdot 36\cdot 36)J = 2\cdot 1296J = 2592J$$

$$L_{OA} = U(A) - U(O) = 2592J$$

Compito D

6.

a)

$$\vec{K}_{palla} = \vec{d} \wedge \vec{p} = \vec{d} \wedge m\vec{v} = 1.2 \cdot 1 \cdot 10 \cdot \frac{1}{2} = 6 kgm^2 s^{-1}$$

b)

$$\vec{K}_{palla} = \vec{K}_{piattaforma+persona}$$
$$\vec{K}_{piattaforma+persona} = 6kgm^2 s^{-1} = I\omega$$
$$I = \frac{1}{2}MR^2 + m_{persona}d^2 = \frac{1}{2}250(1.5)^2 + 85 \cdot 1.44 = 125 \cdot 2.25 + 122.4 = 403.65kgm^2$$
$$\omega = \frac{6}{403.65} = 0.0149 rad / s$$

Compito D

7.

$$0 = m_t v_t - m_r v_{0x}$$

$$v_t = \frac{m_r}{m_t} v_r$$

$$-\frac{1}{2} g t^2 + v_{0y} t = 0$$

$$t_1 = 0$$

$$-\frac{1}{2} g t + v_{0y} = 0$$

$$t_2 = \frac{2 v_{0y}}{g}$$

$$x_r = v_{0x} t_2 = \frac{2 v_{0x} v_{0y}}{g}$$

$$x_t = v_t t_2 = \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g}$$

$$x_r + x_t = l$$

$$l = \frac{2 v_{0x} v_{0y}}{g} + \frac{m_r}{m_t} \frac{2 v_{0x} v_{0y}}{g} = \frac{2}{g} v_0^2 \sin 15° \cos 15° \left(1 + \frac{m_r}{m_t}\right)$$

$$v_0^2 = \frac{g l}{2 \sin 45° \cos 45°} \frac{m_t}{m_t + m_r} = \frac{9.8 \cdot 2.5}{2 \cdot 0.5} \frac{3}{3.2} = 22.97$$

$$v_0 = 4{,}79\, m s^{-1}$$