Problema n. 3

a) $[K_1]\cdot[l^2]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_1]=[m\cdot l^{-1}\cdot t^{-2}]$

$$[K_2]\cdot[l^2]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_2]=[m\cdot l^{-1}\cdot t^{-2}]$$

$$[K_3]\cdot[l^0]=[m\cdot l\cdot t^{-2}] \quad\Rightarrow\quad [K_3]=[m\cdot l\cdot t^{-2}]$$

b) $\vec{\nabla}\wedge\vec{F}=\vec{0} \quad\Rightarrow$

$$\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}=0,\quad \frac{\partial F_y}{\partial z}-\frac{\partial F_z}{\partial y}=0,\quad \frac{\partial F_x}{\partial y}-\frac{\partial F_y}{\partial x}=0$$

c) $\vec{F}=\vec{\nabla}U$

$$F_x=\frac{\partial U}{\partial x},\quad F_y=\frac{\partial U}{\partial y},\quad F_z=\frac{\partial U}{\partial z}$$

$$U(x,y,z)=\int\vec{F}\cdot d\vec{r}=\int\left(-K_1x^2\hat{i}-K_2y^2\hat{j}+K_3\hat{k}\right)\left(dx\hat{i}+dy\hat{j}+dz\hat{k}\right)=\int\left(-K_1x^2dx-K_2y^2dy+K_3dz\right)$$

$$U(x,y,z)=-\frac{1}{3}K_1x^3-\frac{1}{3}K_2y^3+K_3z$$

d) $U(O)=U(0,0,0)=0$

$$U(B)=U(-1,-2,1)=8J$$

$$L_{OB}=U(B)-U(O)=8J$$

Problema n. 4

Forze che agiscono sulla ruota:

$$\vec{F}_R = \vec{F}_A + \vec{F}_T + \vec{F}_C$$

$$\vec{F}_A = \vec{F} + \vec{T}$$
$$\vec{F}_T = -m\vec{a}_0$$
$$\vec{F}_C = 0$$

Equazioni cardinali della dinamica:

$$\begin{cases} F - T - ma_0 = ma \\ TR = I\dot{\omega} \end{cases}$$

$$\dot{\omega} = \frac{\|\vec{a}\|}{R} = \frac{a}{R}$$

$$TR = I\frac{a}{R}$$

$$T = \frac{1}{2}mR^2\frac{a}{R} = \frac{1}{2}ma$$

$$F - \frac{1}{2}ma - ma_0 = ma$$

$$\frac{3}{2}a = \frac{F}{m} - a_0 \quad \Rightarrow \quad a = \frac{2}{3}\left(\frac{F}{m} - a_0\right) \quad \Rightarrow \quad a = \frac{2}{3}\left(\frac{20}{2} - 1.5\right) = 5.6\overline{7}\,ms^{-2}$$

Problema n. 5

a)

$$\frac{1}{2}\cancel{m}gh_0 = \cancel{m}v_i^2$$

$$v_i^2 = 2gh_0$$

$$v_i = \sqrt{2gh_0} = \sqrt{2\cdot 9.8\cdot 0.5} = \sqrt{9.8} = 3.13ms^{-1}$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_i^2 - L_{T_r}$$

$$\vec{T}_r = -\mu\vec{N} = -\mu mg$$

$$L_{T_r} = \mu mg\int_0^d dx = \mu mgx\Big|_0^d = \mu mgd$$

$$\frac{1}{2}\cancel{m}v_A^2 = \frac{1}{2}\cancel{m}v_i^2 - \mu\cancel{m}gd$$

$$v_A^2 = v_i^2 - 2\mu gd$$

$$v_A = \sqrt{v_i^2 - 2\mu gd} = \sqrt{9.8 - 2\cdot 0.4\cdot 9.8\cdot 1} = \sqrt{9.8 - 7.8} = \sqrt{1.96} = 1.4ms^{-1}$$

b)

Conservazione della componente orizzontale della quantità di moto:

$$Q_i = MV_i - mv_A = -mv_A$$

$$Q_f = -MV_f + mv_f = -MV_f$$

$$\Delta Q = Q_f - Q_i = 0 \quad \Rightarrow \quad Q_f = Q_i$$

$$mv_A = MV_f \quad \Rightarrow \quad V_f = \frac{m}{M} v_A$$

Conservazione dell'energia meccanica:

$$T_i = \frac{1}{2} MV_i^2 + \frac{1}{2} mv_A^2 = \frac{1}{2} mv_A^2$$

$$U_i = Mg \cdot 0 + mg \cdot 0 = 0$$

$$T_f = \frac{1}{2} MV_f^2 + \frac{1}{2} mv_f^2 = \frac{1}{2} MV_f^2 = \frac{1}{2} M \frac{m^2}{M^2} v_A^2 = \frac{1}{2} \frac{m^2}{M} v_A^2$$

$$U_f = Mg \cdot 0 + mg \cdot h = mgh$$

$$\Delta E = E_f - E_i = 0 \quad \Rightarrow \quad E_f = E_i \quad \Rightarrow \quad T_f + U_f = T_i + U_i$$

$$\frac{1}{2} \frac{m^2}{M} v_A^2 + mgh = \frac{1}{2} mv_A^2$$

$$h = \frac{1}{2g} \left(1 - \frac{m}{M}\right) v_A^2 = \frac{1}{2 \cdot 9.8} \left(1 - \frac{1}{3}\right) \cdot 1.96 = 0.067m$$

c) $$t = t_d + t_p = t_d + \frac{T_p}{4}$$

$$\mu g = a_{media} = \frac{v_i - v_A}{t_d}$$

$$t_d = \frac{v_i - v_A}{\mu g} = \frac{3.13 - 1.4}{0.4 \cdot 9.8} = 0.44s$$

$$t_d = \frac{v_i - v_A}{\mu g} = \frac{3.13 - 1.4}{0.4 \cdot 9.8} = 0.44s$$

$$T_p = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{8}{9.8}} = 5.68s$$

$$t = 0.44 + \frac{5.68}{4} = 1.86s$$